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I. INTRODUCTION

So far, we have mainly looked at the magnetic behaviour of bulk materials. When the dimensions of a (ferro)magnet get down to the order of micrometres and nanometres, its behaviour may become quite different from bulk. The apparent saturation magnetisation, for example, can become an order of magnitude lower. Around 1950, there still was doubt among physicists about the exact origins of this effect[3], but since then many experiments have shaped and confirmed present theory. This chapter provides an introduction to the physics at micro- and nanometre scale, and shows some recent applications. A more thorough treatment of the theory is given by many textbooks; [2, §11][9, §3.2][5, §6] among others. One noteworthy article is [4, §6] by Kittel.

II. SOME PHYSICS

A. Single- and multi-domain particles

A permanent magnet creates a magnetic field in space, which carries energy. This energy can be minimised by creating oppositely facing domains in the material, so that the external field is decreased. The boundary between domains carries energy, but this is less than the energy benefit obtained. When the dimensions of a magnetic particle get close to the domain wall size, it may be more energetically favourable for the particle to form a single domain.

The critical size between a single- or multi-domain particle can be estimated by considering the two situations depicted in figure 1. The energy of a single-domain par-



Figure 1: Magnetisation of (a) a single domain particle, and (b) the simplest multi-domain particle.

ticle (a) just has the magnetostatic energy of a spherical particle with radius R, saturation magnetisation M_{sat} and demagnetisation factor \mathcal{N} :

$$E_{sd} = \frac{1}{2}\mu_0 \mathcal{N} M_{sat}^2 \cdot \frac{4}{3}\pi R^3.$$
 (1)

The two-domain particle (b) includes the energy of a domain wall:

$$E_{md} = \gamma \pi R^2 + \alpha \frac{1}{2} \mu_0 \mathcal{N} M_{sat}^2 \cdot \frac{4}{3} \pi R^3,$$
 (2)

with γ the domain wall energy per surface area. The exact magnetostatic energy of a two-domain particle is not easily derivable, therefore it is written as a fraction α of the single-domain case. Since the two-domain particle has a lower stray field, the magnetostatic two-domain energy is less than the single-domain energy and $\alpha < 1$. It is obvious that $\alpha > 0$. We'll take $\alpha = \frac{1}{2}$, as is done is several textbooks [2, §9.5][9, §3.2.2.3][5, §6.2.4.3].

The critical domain diameter D_{do} is found when the energies of the single- and multi-domain configuration are equal, $E_{sd} = E_{md}$. This results in

$$D_{do} = 2R_{do} = \frac{6\gamma}{2(1-\alpha)\mu_0 \mathcal{N} M_{sat}^2} = \frac{18\gamma}{\mu_0 M_{sat}^2}.$$
 (3)

Spherical particles with a diameter below D_{do} consist of a single magnetic domain, larger particles consist of multiple domains.

B. Homogeneous rotation

A typical single domain particle will have all its spins pointing in the same direction. When an external magnetic field is applied, these spins rotate in unison to become parallel to the field and minimise the Zeeman energy. This behaviour is also termed cohorent or homogeneous rotation. There may be, however, an anisotropy barrier to be overcome. The external field needed, H_{sw} , to rotate the magnetisation of a spherical particle from opposite to parallel to the field, can be estimated using Stoner-Wohlfarth[10] theory. This has been treated in §4.3 and yields:

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$$H_{sw} = \frac{2K}{\mu_0 M_{sat}}.$$
(4)

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С. The superparamagnetic limit

When the size of a particle is very small, thermal energy $k_B T$ may be enough to overcome the anisotropy barrier KV. This results in the magnetisation being rotated spontaneously every once in a while. With no external field, this yields an average magnetisation of zero, with the averaging done either over time or over multiple particles. When an external field is present, the particle does have an averaged magnetisation parallel to the field. This phenomenon is called superparamagnetism; D_{th} is the diameter below which this occurs, the superparamagnetic limit. The approach of thermally activated switching of a single-domain particle is called the *Neel-Brown* model.

The average time τ_m it takes for the thermal energy to flip the magnetisation can be expressed using the Arrhenius-Neel law[7]:

$$\tau_m = \tau_0 e^{\frac{KV}{k_B T}},\tag{5}$$

with τ_0 the resonance relaxation time of the spin system, which is generally around the order of $10^{-10}s$ [1]. This equation can be understood directly by recognising the Boltzmann-factor, although it is possible to derive it from a magnetic energy expression like done in previous paragraphs [9, §3.4.3.4].

For a given time span τ_m , the critical diameter is

$$D_{th} = \sqrt[3]{\frac{6k_BT}{\pi K} \ln \frac{\tau_m}{\tau_0}}; \tag{6}$$

a particle of this diameter has a stable magnetisation for a time span of just τ_m . The value of τ_m is, of course, application dependent. In superparamagnetic beads for biomedical applications (see §IIIB), a second is appropriate. Hard disks need permanently magnetic areas for storing data and mustn't be disturbed by thermal processes; a lifetime in the order of tens of years is much more reasonable there. The superparamagnetic limit is a real issue for the current generation of hard disks, since magnetic bits cannot grow smaller than that without losing data.

The different regimes D.

So far, we have seen three regimes: multi-domain (bulk, $D > D_{do}$), stable single-domain ($D_{th} < D < D_{do}$), and superparamagnetic $(D < D_{th})$. For homogeneous rotation, the coercive field was determined (equation 4). This is the external field needed to rotate the magnetisation, and a measure for the internal energy.

In figure 2, the coercive field is shown as a function of size. Multi-domain particles decrease the field by forming domains, while single-domain particles have a high field, originating from magnetocrystalline $(2K_1/M_s)$ and

of diameter. The different size regimes can be recognised. [5, figure 6.12]

shape $((N_1 - N_{||}) \cdot M_{sat})$ anisotropy. Below the superparamagnetic limit D_{th} , the coercive field decreases because

thermal energy is enough to overcome the barrier.

Below the critical domain diameter D_{do} , the energy of a domain wall is too large to reduce the stray field. But there are other options for reduction, besides creating multiple domains. This is what happens in the curling and buckling magnetisation modes: the magnetisation various continuously over the geometry. The exchange energy resulting from slightly nonparallel spins is kept small, while spins are cancelling each others field over a larger area. Figure 3 shows the most basic modes. We have seen homogeneous rotation (a) before, where all spins are parallel. With buckling (b), magnetisation is homogeneously varying along one direction. In the figure, it is varying upwards, and fluctuations to the left and right cancel each other, thereby decreasing the total stray field. Buckling often occurs in thin rods.

Curling or vortex (c) is an interesting mode, where the stray field is reduced to a minimum by circular symmetry. It has also small exchange energy, but for the centre: there opposed spins are just next to each other, forcing the magnetisation to rotate out of plane. This mode is found primarily in thin disks, with possible applications in data storage (§IIIC); this vortex state has a centre magnetisation either into the paper or out of it.

Figure 2: A schematic graph of the coercive field as a function



Figure 3: Magnetisation modes of a single domain: (a) homo-

geneous rotation, (b) buckling, and (c) curling. The centre of the curling mode is pointing out of (or into) the plane.



Figure 4: Experimental relation between diameter and coercive field for particles deriving their coercive field principally from crystal anisotropy energy. Measured at a temperature of 76K. Adapted from [6, figure 2], where data was collected from various sources. Theoretical critical diameters as derived here are indicated on top.

E. Experimental confirmation

Theory can only go as far as experiments allow it to. The last century has indeed seen a multitude of experiments on magnetic behaviour of small spheres, rods and layers; and this still an active area of research today.

Figure 4 shows some of these results for iron and cobalt. The coercive field was measured as a function of particle diameter, producing a graph like figure 2 from experimental data. On top, critical diameters as derived previously are indicated. The superparamagnetic limit D_{th} seems to correspond well with theory. The critical domain diameter D_{do} may be correct for cobalt, but certainly is too small for iron. This may have been caused, for example, by shape-anisotropy from particles that weren't exactly spherical after all. The exact shape is also important in the formation of curling and buckling modes, which aren't recognisable here. Section III C has an experimental example of the curling mode.

All in all, the relatively basic theory presented here appears to be able to give a good initial estimation of magnetic behaviour on a micrometre and nanometre scale.

III. APPLICATIONS

A. Ferrofluids

Certain superparamagnetic particles can be dissolved into a fluid. When a magnetic field is applied to the fluid, the particles will respond and drag the fluid along, creating a magnetic liquid: ferrofluid (see figure 5a). In a true ferrofluid, particles are perfectly dissolved and no clusters of multiple particles exist. Since superparamag-



Figure 5: (a) A ferrofluid is a solution of superparamagnetic particles in a liquid, effectively creating a superparamagnetic liquid; source: technorama. (b) A magnetic bead, consisting of multiple superparamagnetic grains.

netic particles have no net magnetisation in zero field, they don't attract each other magnetically. To minimise other attractive forces and to avoid clustering when a magnetic field is present, particles are generally coated with a repelling surfactant. This limits the lifetime of most ferrofluids to a few years.

Ferrofluids have many applications. They are used, for example, as a liquid seal around the spinning drive shaft in a hard disk, one that doesn't wear out. Ferrofluids appear in radar absorbent paint to create stealth aircrafts. Another important application is in microlitre-range fluid manipulation or lab-on-a-chip systems. A small amount of ferrofluid, driven by a magnetic field, can be used to push another fluid forward and create a ferrofluidic micropump[8].

B. Biological assays

A regular task in molecular biology or biomedics is to detect certain biological molecules, for example in blood. A common method is to use magnetic particles (called beads) of about a micrometre in diameter coated with anti-bodies that bind to the specific molecule. The magnetic particles can be washed out of the liquid by a magnet, after which the attached molecules can be detected conveniently.

The magnetic beads are superparamagnetic. This avoids magnetic interaction in zero field (which is convenient, for example, when working with metal tools) and gives a high susceptibility. But a micrometre is by far above the superparamagnetic limit, so these beads are made of a polymer matrix with smaller superparamagnetic grains inside (see figure 5b). The distance between these grains is large enough to avoid exchange interaction, so single-domain superparamagnetic behaviour is retained.



Figure 6: MFM observation of electrical switching of a vortex core. (a) An AFM image of the sample. A permalloy disc fills the white circle. Two wide Au electrodes, through which an a.c. excitation current is supplied, are also seen. (b) MFM image before the application of the excitation current. A dark spot at the centre of the disk inside the red circle indicates that the core magnetisation points upwards with respect to the paper plane. (c) MFM image after the application of the a.c. excitation current. The dark spot at the centre of the disk changed to a bright spot, indicating the switching of the core magnetisation from up to down. [11, figure 3]

C. Vortex switching

A final, recent example demonstrates the existence of the curling or vortex mode, mentioned in section IID. A permalloy disk with a thickness of 50nm and a radius of 500nm was imaged using MFM, shown in figure 6. The disk has a curling mode, with a vortex in its centre pointing either up- or downwards. The article by Yamada et al. [11] uses an a.c. excitation current through the disk to switch the direction of the vortex's magnetisation. Magnetic force measurements (MFM) show that before application of the current, the central spin pointed upwards in figure 6b; after the excitation current, the spin was found to be pointing downwards, as can be seen in figure 6c. This magnetic vortex structure that is switched by an electrical current, may become a building block for spintronic memory devices of the future.

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